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Modeling Volatility in Foreign Currency Option Pricing*

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Abstract

This paper presents a general optimization framework to forecast put and call option prices by exploiting the volatility of the options prices. The approach is flexible in that different objective functions for predicting the underlying volatility can be modified and adapted in the proposed framework. The framework is implemented empirically for four major currencies, including Euro. The forecast performance of this framework is compared with the forecast performance of the Multiplicative Error Model (MEM) of implied volatility and the GARCH(1,1). The results indicate that the proposed framework is capable of producing reasonably accurate forecasts for put and call prices.

Keywords: foreign currency options, implied volatility, optimal volatility, multiplicative error model, GARCH model

JEL classification: G12, G13

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1. Introduction

The well-known Black-Scholes (1973) option pricing model (BS) provides the foundation for pricing of options and derivatives. Unfortunately, BS does not evaluate the market's expectation of future volatility, but the expectation can be obtained by inverting the observed option price. For each observed option price, the implied volatility (IV) is the volatility implied by the BS option pricing formula given the observed price. This IV is widely believed to be the market's best forecast regarding the future volatility over the remaining life of the option. However, IV may be a biased representation of market expectations for the following reasons: (i) transaction prices may not represent equilibrium market prices; (ii) the option pricing model may be specified incorrectly; and (iii) as the volatility of asset returns tends to change over time, the constant variance assumption may be unrealistic.

A number of studies have focused on the predictive power of IV. The empirical results are at best mixed. Earlier research by Latane and Rendleman (1976), Schmalensee and Trippi (1978), Chiras and Manaster (1978), Beckers (1981) indicated that that IV was a better predictor of actual volatility than volatility based on historical data. Lamourex and Lastrapes (1993) conducted a joint test of the Hull-White (1987) option pricing model and market efficiency, and they find that although IV helps predict volatility, available information in historical data can be used to improve the market's forecasts as measured by IV. Day and Lewis (1992) show that IV in the equity market contains incremental information relative to the conditional volatility from GARCH models. Similar results are also reported in Fleming et al. (1995), Christensen and Prabhala (1998), Fleming (1998), Bates (2000), and Kazantzis and Tessaromatis (2001). In contrast, Canina and Figlewski (1993) find that IV volatility has little predictive power for future volatility. Jorion (1995), however, reports that IV outperforms statistical time-series models in terms of information content and predictive power, but IV appears to be too variable relative to future volatility.

Harvey and Whaley (1992), using S&P 100 index option, report that implied volatility changes can be predicted ahead of time. This study also indicates that implied volatilities tend to fall on Fridays and rise on Mondays. Using CBOE Market Volatility Index (VIX), an average of S&P 100 option implied volatilities, Fleming et al. (1995), however, rejects inter-week seasonality. Furthermore, this study indicates that VIX is inversely related to the contemporaneous S&P 100 index return, and that both daily and weekly VIX changes are more sensitive to the negative than the positive stock market moves. Simon (1997) also reports similar implied volatility asymmetries for treasury bonds and futures options. Ederington and Lee (1996), however show that the implied volatilities in the treasury bonds and Eurodollar options on futures markets tend to decline on the days with scheduled macroeconomic announcements, which are also responsible for the inter-week patterns of implied volatilities.

As widely known, BS is mainly used for valuing options on stocks. This model has also spawned the field of financial engineering, which is dedicated to designing and implementing such derivatives pricing models. BS assumes that no dividends are paid on the stock during the life of the option. This model is extended by Merton (1973) for continuous dividends. Since the interest gained on holding a foreign security is equivalent to a continuously paid dividend on a stock share, the Merton version of the BS can be applied to foreign security. To value currency option, stock prices are substituted for exchange rates.

The first application of modern valuation techniques to currency options is generally credited to Grabbe (1983) and Garman and Kohlhagan (1983). They considered foreign currency as an asset and expected returns from holding foreign currency depend on the volatility of exchange rate in their model. The practical relevance of this model as an approximate currency options pricing formula depends on the investor's ability to forecast exchange rate variability over the remaining life of the option. The model is however, based on several standard assumptions.

This paper provides a new approach to measuring volatility of currency options prices explicitly from their past history. A general optimization framework is proposed to forecast put and call option prices by constructing optimal volatility forecasts based on past information. The volatility is calculated as the weighted sum of the past squared returns by minimizing the in-sample mean squared errors between market and model prices. The future prices are predicted using the BS option pricing model given the volatility forecasts. The objective is to assess how well the past options market prices would forecast the future ones. The emphasis is on assessing the accuracy of the forecasts, rather than on how forecasts are formed.

The paper has several attractive features. First, unlike other approaches in the literature, this paper is concerned with modeling volatility as an instrument to predict future option prices, rather as a measure of risk. Second, this paper proposes a general framework to forecast future option prices. This framework is flexible as it can be modified to accommodate different objective functions to forecast future volatility with different option pricing models. Thirdly, this paper uses the past option prices, rather than the underlying currency prices, to calculate volatility. Although options derive their values from the underlying currencies, spot and options markets are treated as separate entities in this framework. This is a new idea in the options literature. Finally, unlike the majority of work focusing on stocks and bonds options, the current paper focuses on options on major currencies, including Euro.

The paper is organized as follows. The next section gives the research methodology and the data used in this study, followed by in-sample fitting and out-of-sample prediction test in sections 3 and 4 respectively. The last section concludes the paper.

2. Methodology and Data

The framework proposed in this paper can be summarized in the following steps. The first step involves selecting a pricing model to generate future prices. Unless otherwise stated, the pricing model chosen in this paper is the BS option pricing model. Although the constant variance assumption underlying the BS seems restrictive in practice, it is not necessarily the case. It is highly possible that the variance is constant over a small time interval but it is time varying over a longer time horizon. In such a case, the BS is a valid model over each of the small time intervals. This paper assumes that the variance of the underlying asset's return may be constant within a small time interval (one day) but changing from one interval to another, that is, variance changes on the daily basis but constant within the day. The implication of this assumption is that IV derived from the BS of a particular day would be a reasonable approximation of the true underlying volatility for that day.

In theory, if IV can be predicted ahead of time with reasonable accuracy, then these volatility forecasts can be used as inputs to the BS option pricing formula to forecast future call and put prices. This is the second step of the proposed framework. Given the pricing formula, volatility could be predicted as a weighted average of the past squared returns, with weights calculated by optimizing appropriate objective functions. The above idea can be implemented as a simple spreadsheet-based application, and we name it as optimal weighted volatility (OV) model. In this approach, the volatility is modeled as a linear combination of the past squared returns from the observed put and call prices, with weights calculated by minimizing a given objective function. This approach also describes the pattern of the volatility which contains important information about investors' behavior over time. Finally, the forecast performance of BS option pricing formula using OV is compared with alternative volatility models including the Multiplicative Error Model (MEM) of IV and GARCH(1,1) model of past squared returns. If OV provides superior volatility for predicting future prices, then this approach will be an innovative way to identify the underlying process of valuing currency options. In what follows, we describe the details of this methodology, and the following notations are used throughout the paper:

S_t spot exchange rate at time t;

T expiration time of the option;

C_t market price of a call option in domestic currency at time t;

P_t market price of a put option in domestic currency at time t;

X_t option exercise price in domestic currency at time t;

R_T^d continuously compounded rate of return on risk-free domestic interest rate with the maturity at time T;

B_t domestic currency price at time t of a risk-free discount bond $\left(e^{-R_t^d T}\right)$ which pays one unit of domestic currency at the expiration time, T;

R_T^f continuously compounded rate of return on risk-free foreign interest rate with the maturity at time T;

B_t^* foreign currency price at time t of a risk-free discount bond $\left(e^{-R_t^f T}\right)$ which pays one unit of foreign currency at the expiration time, T;

N cumulative normal distribution function;

σ_t volatility of the exchange rate at time *t*.

The price of a European call option on currency is stated as,

$$C_t = S_t B_t^* N\left(d_{t,1}\right) - X_t B_t N\left(d_{t,2}\right), \tag{1}$$

Similarly, the price of a European put option on currency is stated as,

$$P_t = X_t B_t N\left(-d_{t,2}\right) - S_t B_t^* N\left(-d_{t,1}\right), \tag{2}$$

where, $$d_{t,1} = \frac{\ln\left(S_t / X_t\right) + \left(R_t^d - R_t^f + \sigma^2/2\right)T}{\sigma_t \sqrt{T}}, \text{ and}$$

$$d_{t,2} = \frac{\ln\left(S_t / X_t\right) + \left(R_t^d - R_t^f - \sigma^2/2\right)T}{\sigma_t \sqrt{T}} = d_{1,t} - \sigma_t \sqrt{T} .$$

For notation convenience, let's define

$$\xi_t = S_t B_t^*, \qquad \eta_t = X_t B_t,$$

and hence, equations (1) and (2) can be rewritten as

$$C_t = \xi_t N\left(d_{t,1}\right) - \eta_t N\left(d_{t,2}\right), \tag{3}$$

$$P_t = \eta_t N\left(-d_{t,2}\right) - \xi_t N\left(-d_{t,1}\right). \tag{4}$$

In equations (3) and (4), all parameters except the volatility are directly observable from market data. This allows a market-based estimate of volatility of a foreign security. A variety of methods is available for estimating the volatility and most researchers use the implied standard deviation (ISD) from option market price as the current estimate of IV. Rewriting equation (3) and (4) yields,

$$f(\sigma_t) = \xi_t N\left[d_{t,1}(\sigma_t)\right] - \eta_t N\left[d_{t,2}(\sigma_t)\right] - C_t, \tag{5}$$

$$g(\sigma_t) = \eta_t N\left[-d_{t,2}(\sigma_t)\right] - \xi_t \left[-d_{t,1}(\sigma_t)\right] - P_t. \tag{6}$$

Thus, the calculation of IV is equivalent to finding σ such that

$$f(\sigma_t) = 0$$
$$g(\sigma_t) = 0.$$

Given both $f(\sigma_t)$ and $g(\sigma_t)$ are highly non-linear functions, the calculation of IV requires numerical procedures such as the Newton-Raphson method. In this paper, a hybrid of Newton-Raphson and Bisection methods are used to calculate IV. The

Newton-Raphson method is an iterative technique based on the first order Taylor expansion of the function. The iterative formula can be written as

$$\sigma_{t,n+1} = \sigma_n - \frac{f(\sigma_{t,n})}{f'(\sigma_{t,n})},$$

for IV derived from the call option. Replacing $f(\sigma_{t,n})$ by $g(\sigma_{n,t})$ yields the iterative formula for IV derived from the put option. Note that the first derivatives of $f(\sigma_{t,n})$ and $g(\sigma_{t,n})$ coincide, that is, $f'(\sigma_{t,n}) = g'(\sigma_{t,n})$.

Now, given

$$\frac{\partial d_{t,1}}{\partial \sigma_t} = \frac{1}{\sigma_t^2 \sqrt{T}} \left[\left(\frac{\sigma^2}{2} - R_t^d + R_t^f \right) T - \log \frac{S_t}{X_t} \right],$$

and

$$\frac{\partial d_{t,2}}{\partial \sigma_t} = \frac{\partial d_{t,1}}{\partial \sigma_t} - \sqrt{T},$$

then the first derivative, $f'(\sigma)$, is

$$f'(\sigma_t) = \frac{1}{\sqrt{2\pi}} \left\{ \sqrt{T} \eta_t \exp\left(\frac{-d_{t,2}^2}{2} \right) + \left[\xi_t \exp\left(\frac{-d_{t,1}^2}{2} \right) - \eta_t \exp\left(\frac{-d_{t,2}^2}{2} \right) \right] \frac{\partial d_{t,1}}{\partial \sigma_t} \right\}$$

for both put and call options.

Although the Newton-Raphson procedure converges quicker than the bisection algorithm, it does have several drawbacks. First, the initial value, $\sigma_{t,0}$, must be fairly close to the solution and second, $f'\left(\sigma_{t,n}\right) \neq 0, \quad \forall n \in Z^+$. In the event when Newton-Raphson failed, the equation can be solved by using the bisection method as follows:

Step 1: Find $\sigma_{t,n}^+$, such that $f\left(\sigma_{t,n}^+\right) > 0$.

Step 2: Find $\sigma_{t,n}^-$, such that $f\left(\sigma_{t,n}^-\right) < 0$.

Step 3: Calculate $\sigma_{t,n} = \dfrac{\left(\sigma_{t,n}^+ + \sigma_{t,n}^-\right)}{2}$.

Step 4: If $f\left(\sigma_{t,n}\right) > 0$ then $\sigma_{t,n+1}^+ = \sigma_{t,n}, \sigma_{t,n+1}^- = \sigma_{t,n}^-$ and if $f\left(\sigma_{t,n}\right) < 0$, then

$\sigma^-_{t,n+1} = \sigma_{t,n}$, $\sigma^+_{t,n+1} = \sigma^+_{t,n}$. If $f\left(\sigma_{t,n}\right) = 0$, then STOP – the solution has been found!

Step 5: If the algorithm was not terminated in Step 4, then repeat Step 1 to 4 until either it is terminated in Step 4 or $\left|f\left(\sigma_n\right)\right| < \tau$ where τ is the pre-determined tolerance level. Unless otherwise stated, $\tau = 10^{-5}$ in this paper.

Given the IV is calculated using the algorithm above, the next step is to forecast future volatility based on this information. One way to forecast future volatility is to estimate the following model for σ_t.

$$\sigma_{i,t} = \varepsilon_{i,t}\sqrt{h_{i,t}}, \quad \varepsilon_{i,t} \sim \text{iid}(1,\upsilon), \quad \varepsilon_{i,t} > 0\ \forall t, \quad i = C, P.$$
$$h_{i,t} = \omega_i + \alpha_i \sigma^2_{i,t-1} + \beta_i h_{i,t-1} \quad \omega, \alpha, \beta > 0.$$

This specification implies that the IV follows a Multiplicative Error Model (MEM) as proposed in Engle (2002). Given the past information, the future volatility is estimated by

$$\hat{\sigma}^{IV}_{i,t} = \sqrt{h_{i,t}} \tag{7}$$

for $\forall_i = C, P$ and hence, the implied volatility model price (IVP) for calls $\left(\hat{\Pi}^{IVP}_{C,t}\right)$ and puts $\left(\hat{\Pi}^{IVP}_{P,t}\right)$ can be generated by

$$\hat{\Pi}^{IVP}_{C,t} = \xi_t N\left[d_{t,1}\left(\hat{\sigma}^{IV}_{C,t}\right)\right] - \eta_t N\left[d_{t,2}\left(\hat{\sigma}^{IV}_{C,t}\right)\right], \tag{8}$$

$$\hat{\Pi}^{IVP}_{P,t} = \eta_t N\left[-d_{t,2}\left(\hat{\sigma}^{IV}_{P,t}\right)\right] - \xi_t N\left[-d_{t,1}\left(\hat{\sigma}^{IV}_{P,t}\right)\right], \tag{9}$$

Note that in MEM presentation as above, the noise sequence $\left\{\varepsilon_{i,t} : t \in N\right\}$ does not need to be specified as we estimate the model by quasi-maximum likelihood estimator (QMLE) with normal density (see Ling and McAleer, 2003).

Another way to forecast volatility is to assume that the return of the prices, $r_{i,t}$ for $\forall_i = C, P$, follow a GARCH(1,1) process, that is,

$$r_{i,t} = \kappa_{i,t}\sqrt{g_{i,t}}, \quad \kappa_{i,t} \sim \text{iid}(0,1) \quad i = C, P$$
$$g_{i,t} = w_i + \alpha_i r^2_{i,t-1} + h_{i,t-1}.$$

Given this specification, the future GARCH (1,1)-based volatility (GV) for both put and call prices are estimated by

$$\hat{\sigma}_{i,t}^{GV} = \sqrt{g_{i,t}} \tag{10}$$

for $\forall_i = C, P$. Although this specification is very similar to the IV approach, the underlying assumptions are quite different. In the IV approach, the MEM specification aims to model the volatility and not the return. Therefore, the independent and identically distributed random variable, ε_t, must have a distribution such that $P(\varepsilon_t < 0) = 0$. However, in the GARCH(1,1) specification above, the endogenous variable is the price returns which can be positive or negative and hence, the iid random variable, $\kappa_{i,t}$, can be any real number. Using estimated GV $\left(\hat{\sigma}_{i,t}^{GV}\right)$, the GARCH (1,1)-based volatility model price (GVP) for calls $\left(\hat{\Pi}_{C,t}^{GVP}\right)$ and puts $\left(\hat{\Pi}_{P,t}^{GVP}\right)$ can be generated as

$$\hat{\Pi}_{C,t}^{GVP} = \xi_t N\left[d_{t,1}\left(\hat{\sigma}_{C,t}^{GV}\right)\right] - \eta_t N\left[d_{t,2}\left(\hat{\sigma}_{C,t}^{GV}\right)\right], \tag{11}$$

$$\hat{\Pi}_{P,t}^{GVP} = \eta_t N\left[-d_{t,2}(\hat{\sigma}_{P,t}^{GV})\right] - \xi_t N\left[-d_{t,1}(\hat{\sigma}_{P,t}^{GV})\right] \tag{12}$$

Again, note that in implementing the GARCH (1,1) model, the noise sequence does not need to be specified as the model is estimated by QMLE.

To compute OV to forecast future call and put option prices, we propose a novel way to utilise previous returns of the option prices. For this purpose, we select an optimal combination of previous absolute returns as a predictor for future volatility and use it as an input to the BS option pricing formula as defined in (1) and (2) to predict future option prices. Thus, to forecast future volatility, we have

$$\hat{\sigma}_t^{OV} = w' r_t \tag{13}$$

where $w' = (w_1, ..., w_q)'$, $w_i \in [0,1]$ $\forall i = 1..q$, such that, $\sum_{i=1}^{p} w_i = 1$, $r_t = (r_{c,t-1}, ..., r_{c,t-l}, r_{p,t-1}, ..., r_{p,t-m})$, $l + m = q$, $r_{c,t}$ and $r_{p,t}$ are the absolute returns of the call and put equation, respectively. Let $\Pi_{C,t}^{MP}$ and $\Pi_{P,t}^{MP}$ denote the call and put observed prices, respectively, at time, t. The returns are then calculated as

$$r_{i,t} = \log\left(\Pi_{i,t}^{MP}\right) - \log\left(\Pi_{i,t-1}^{MP}\right), \quad \forall i = C, P.$$

Denote OV model price by $\Pi_{i,t}^{OV}$, where $i = C, P$. The weight vector, w, can be chosen to minimize the with-in-sample mean squared error (MSE) between $\Pi_{i,t}^{MP}$ and $\Pi_{i,t}^{OV}$:

$$w = \min_{\omega} \left[\left(\Pi_{P,t}^{MP} - \Pi_{P,t}^{OV} \right)^2 + \left(\Pi_{C,t}^{MP} - \Pi_{C,t}^{OV} \right)^2 \right]. \tag{14}$$

Similarly, for mean absolute error (MAE) between $\Pi_{i,t}^{MP}$ and $\Pi_{i,t}^{OV}$, we have

$$w = \min_{\omega} \left[\left| \Pi_{P,t}^{MP} - \Pi_{P,t}^{OV} \right| + \left| \Pi_{C,t}^{MP} - \Pi_{C,t}^{OV} \right| \right], \tag{15}$$

and for mean absolute percentage error (MAPE) between $\Pi_{i,t}^{MP}$ and $\Pi_{i,t}^{OV}$, we have

$$w = \min_{\omega} \left[\left| \frac{\Pi_{P,t}^{OV} - \Pi_{P,t}^{MP}}{\Pi_{P,t}^{MP}} \right| + \left| \frac{\Pi_{C,t}^{OV} - \Pi_{C,t}^{MP}}{\Pi_{C,t}^{MP}} \right| \right], \tag{16}$$

where ω is an arbitrary vector of weights giving the combination that minimizes the objective function. Note that the optimal weight vector (w) is constant over time.

Now the optimal weighted volatility model price for calls $\left(\hat{\Pi}_{C,t}^{OVP} \right)$ and puts $\left(\hat{\Pi}_{P,t}^{OVP} \right)$ can be generated by

$$\hat{\Pi}_{C,t}^{OVP} = \xi_t N\left[d_{t,1}\left(\hat{\sigma}_t^{OV} \right) \right] - \eta_t N\left[d_{t,2}\left(\hat{\sigma}_t^{OV} \right) \right] \tag{17}$$

$$\hat{\Pi}_{P,t}^{OVP} = \eta_t N\left[-d_{t,2}\left(\hat{\sigma}_t^{OV} \right) \right] - \xi_t N\left[-d_{t,1}\left(\hat{\sigma}_t^{OV} \right) \right] \tag{18}$$

Note that this method utilizes the past information provided by both call and put prices. Also it is to be noted that in implementing this procedure, our data on calls and puts have the same time to maturity. However, they do not have the same moneyness; when a call is in ITM, the corresponding put is OTM, as the call-put pairs have the same strike prices. Unless otherwise stated, the solutions to the optimization problem as stated in equations (14) to (16) are obtained by using the Solver™ application in Microsoft Excel™ with the default Newton algorithm. Thus, this analysis can be conducted without any additional programming and it would be more suitable as a practical application.

We compute in-sample pricing errors to check for goodness-of-fit, and out-of-sample pricing errors to check for predictive power. Pricing error is defined as the deviation of model price from the observed market price. If $\Pi_{i,t}^{MP}$ and $\hat{\Pi}_{i,t}^{j}$ is the observed market price and estimated model price, respectively, we have the following criteria to measure the pricing errors to assess the forecasting performance of the models:

(i) The mean squared error (MSE) = $\frac{1}{s}\sum_{t=1}^{s}\left(\Pi_{i,t}^{MP} - \hat{\Pi}_{i,t}^{j}\right)^2$,

(ii) The mean absolute error (MAE) = $\frac{1}{s}\sum_{t=1}^{s}\left|\Pi_{i,t}^{MP} - \hat{\Pi}_{i,t}^{j}\right|$,

(iii) The mean absolute percentage error (MAPE) = $\frac{1}{s}\sum_{t=1}^{s}\left|\frac{\hat{\Pi}_{i,t}^{j} - \Pi_{i,t}^{MP}}{\Pi_{i,t}^{MP}}\right|$,

for $\forall i = C, P$ and $\forall j = IVP, GVP, OVP$. The estimated errors are labeled as IVPE, GVPE and OVPE for the three models, respectively. We now proceed to apply the foregoing methodology to the data.

The Data

The data used in this paper are for the following four currency options – the British pound, the Euro, the Japanese yen, and the Swiss franc. All data are obtained from DATASTREAM database, and provided in a separate appendix available on request. The data consist of daily closing prices for each option traded on the PHLX, daily spot exchange rates, and daily Eurocurrency interest rates for the period. Option on Euro started trading December 2000. The data set for all currencies, therefore, includes the options trading period from January 2001 to March 2006. There are some inconsistent data (due to recording error in the database) for the Japanese yen from January 2001 to end of March 2001 and consequently, these are excluded from the sample. The total number of put-call pairs from the observations of daily prices across all four currencies is 5377. The expiration dates of options are within 90 days during the sample period. If the expiration month has 5 Fridays, the options expire on the third Friday, otherwise second Friday of the expiration month. The Eurocurrency interest rates are used to determine daily domestic and foreign bond prices, respectively.

3. The In-Sample Fit

This section presents the in-sample empirical results. For in-sample tests, the implied volatility model pricing error (IVPE) and optimal weighted volatility model pricing error (OVPE) are estimated under the three objective functions (MSE, MAE, MAPE). The results under MSE as the objective function [using equation (14)] are given in Table 1. As can be seen, the OV model outperforms the IV model based on MSE for option prices of all four currencies. Under MSE, OVPE is less than IVPE, on average, by 70.55 percent for British pound, 52.91 percent for Euro, 70.95 percent for Japanese yen and 67.23 percent for Swiss franc. It indicates that OV model prices fit the in-sample market prices better than those from the IV model. However, the MAE and MAPE results in Table 1 are not favorable to OV model. These two measures indicate that IV model tends to do better than the OV model. Interestingly, very similar results can be observed in Table 2 and Table 3 in which MAE and MAPE were used as objective functions, respectively.

The OVPE results reported in Table 1 are based on OV which is computed by the weights that captured random information from past five options trading day options prices. We now explore the nature of these weights for each option price of each currency over the previous five trading days. The observed weights, under MSE as the objective function, are given in Table 4. As can be seen from the last column of the table, over a five-day window, the total weights of call price volatility is higher than the total weights of put price for British pound and Swiss franc. For Euro, the total weights of put price volatility are higher than the total weights of call price volatility. These weights are somewhat evenly distributed for Japanese yen between calls and puts. Thus, these weights do not seem to follow any systematic pattern across currencies. However, it is to be noted that the options on all four sample currencies are traded against the U.S. dollar in the U.S. market. Since the trading volume affects volatility, the relatively higher weights of call price volatility may indicate that trading volume of call options on British pound and Swiss franc has been higher than that of put options. This might imply that the U.S. market is a net importer in British pound and Swiss franc denominated goods and services over the sample period. Similarly, higher weights of put price volatility may indicate that the U.S. market is a net exporter in Euro denominated goods and services. With MAE and MAPE as objective functions, very similar results can be observed, as reported in Table 5 and Table 6, respectively. Thus, the weight function does not seem to be sensitive to the choice of the objective function.

Table 1: Comparison of OVPE and IVPE (In-Sample): MSE Objective Function

Measures	Currency	Options	OVPE	IVPE	$\frac{\text{OVPE} - \text{IVPE}}{\text{IVPE}}\%$	Average difference %
MSE	British pound	Call	0.0804	0.2322	-65.37	-70.55
		Put	0.1050	0.4327	-75.73	
	Euro	Call	0.0813	0.1655	-50.88	-52.91
		Put	0.1107	0.2457	-54.95	
	Japanese yen	Call	0.0396	0.1485	-73.33	-70.95
		Put	0.0314	0.0999	-68.57	
	Swiss franc	Call	0.0368	0.0979	-62.41	-67.23
		Put	0.0298	0.1066	-72.05	
MAE	British pound	Call	0.6977	0.3556	96.20	84.16
		Put	0.7916	0.4599	72.12	
	Euro	Call	0.6639	0.3267	103.21	100.26
		Put	0.7452	0.3777	97.30	
	Japanese yen	Call	0.4585	0.3572	28.36	39.23
		Put	0.4473	0.2980	50.10	
	Swiss franc	Call	0.4345	0.2848	52.56	67.56
		Put	0.4283	0.2346	82.57	
MAPE	British pound	Call	0.6149	0.2639	133.00	110.44
		Put	0.4864	0.2589	87.87	
	Euro	Call	0.7082	0.3291	115.19	117.12
		Put	0.6350	0.2899	119.04	
	Japanese yen	Call	0.4436	0.3975	11.60	27.19
		Put	0.4943	0.3462	42.78	
	Swiss franc	Call	0.5224	0.3265	60.00	65.67
		Put	0.5087	0.2969	71.34	

(Columns OVPE through Average difference % fall under the spanning header "Model pricing errors and their difference in percentage".)

Notes: OVPE and IVPE represent optimal weighted volatility model pricing error and implied volatility model pricing error, respectively. MSE and MAE measures are to be divided by 1000 and 100, respectively. In the last column, the average negative and positive differences indicate that OVPE is less than IVPE and OVPE is more than IVPE, respectively, by reported percent.

Table 2: Comparison of OVPE and IVPE (In-Sample): MAE Objective Function

Measures	Currency	Options	Model pricing errors and their difference in percentage			
			OVPE	IVPE	$\frac{\text{OVPE} - \text{IVPE}}{\text{IVPE}}\%$	Average difference %
MSE	British pound	Call	0.0808	0.2322	-65.20	-70.46
		Put	0.1051	0.4327	-75.71	
	Euro	Call	0.0912	0.1655	-44.89	-48.03
		Put	0.1200	0.2457	-51.16	
	Japanese yen	Call	0.0400	0.1485	-73.06	-70.77
		Put	0.0315	0.0999	-68.47	
	Swiss franc	Call	0.0368	0.0979	-62.41	-67.23
		Put	0.0298	0.1066	-72.05	
MAE	British pound	Call	0.6976	0.3556	96.18	83.98
		Put	0.7900	0.4599	71.78	
	Euro	Call	0.6499	0.3267	98.93	95.84
		Put	0.7280	0.3777	92.75	
	Japanese yen	Call	0.4578	0.3572	28.16	38.80
		Put	0.4453	0.2980	49.43	
	Swiss franc	Call	0.4338	0.2848	52.32	67.42
		Put	0.4282	0.2346	82.52	
MAPE	British pound	Call	0.6169	0.2639	133.76	110.64
		Put	0.4855	0.2589	87.52	
	Euro	Call	0.7005	0.3291	112.85	112.14
		Put	0.6129	0.2899	111.42	
	Japanese yen	Call	0.4432	0.3975	11.50	26.88
		Put	0.4925	0.3462	42.26	
	Swiss franc	Call	0.5222	0.3265	59.94	65.55
		Put	0.5082	0.2969	71.17	

Notes: OVPE and IVPE represent optimal weighted volatility model pricing error and implied volatility model pricing error, respectively. MSE and MAE measures are to be divided by 1000 and 100, respectively. In the last column, the average negative and positive differences indicate that OVPE is less than IVPE and OVPE is more than IVPE, respectively, by reported percent.

4. Out-of-sample Prediction Test

In-sample results, in general, indicate that OV model outperforms IV model for pricing options under the MSE measure for all three objective functions. One may, however, argue that the OV model price fits in-sample better due to the additional explanatory power from higher degrees of freedom. As a check, the out-of-sample predictive power of the OV model is now examined in this section. For this purpose, the predictive power of OV model is assessed against GV model, and the IV model under MEM.

Table 3: Comparison of OVPE and IVPE (In-Sample): MAPE Objective Function

Measures	Currency	Options	Model pricing errors and their difference in percentage			
			OVPE	IVPE	$\frac{\text{OVPE} - \text{IVPE}}{\text{IVPE}}\%$	Average difference %
MSE	British pound	Call	0.0809	0.2322	-65.16	-70.25
		Put	0.1067	0.4327	-75.34	
	Euro	Call	0.0943	0.1655	-43.02	-46.48
		Put	0.1230	0.2457	-49.94	
	Japanese yen	Call	0.0400	0.1485	-73.06	-70.77
		Put	0.0315	0.0999	-68.47	
	Swiss franc	Call	0.0369	0.0979	-62.31	-67.13
		Put	0.0299	0.1066	-71.95	
MAE	British pound	Call	0.6999	0.3556	96.82	85.25
		Put	0.7987	0.4599	73.67	
	Euro	Call	0.6510	0.3267	99.27	96.07
		Put	0.7285	0.3777	92.88	
	Japanese yen	Call	0.4577	0.3572	28.14	38.80
		Put	0.4454	0.2980	49.46	
	Swiss franc	Call	0.4342	0.2848	52.46	67.62
		Put	0.4288	0.2346	82.78	
MAPE	British pound	Call	0.6036	0.2639	128.72	109.38
		Put	0.4920	0.2589	90.03	
	Euro	Call	0.7004	0.3291	112.82	111.97
		Put	0.6120	0.2899	111.11	
	Japanese yen	Call	0.4430	0.3975	11.45	26.85
		Put	0.4925	0.3462	42.26	
	Swiss franc	Call	0.5201	0.3265	59.30	65.37
		Put	0.5090	0.2969	71.44	

Notes: OVPE and IVPE represent optimal weighted volatility model pricing error and implied volatility model pricing error, respectively. MSE and MAE measures are to be divided by 1000 and 100, respectively. In the last column, the average negative and positive differences indicate that OVPE is less than IVPE and OVPE is more than IVPE, respectively, by reported percent.

Table 4: Weights Vector for Full Sample: MSE Objective Function

Currency	Options (i)	Weights corresponding to previous day 1 to day 5					Total Weight
		$w_{i,t-1}$	$w_{i,t-2}$	$w_{i,t-3}$	$w_{i,t-4}$	$w_{i,t-5}$	
British pound	Call	0.1094	0.1258	0.1182	0.1275	0.1462	0.6271
	Put	0.0862	0.0639	0.0665	0.0724	0.0839	0.3729
		0.1956	0.1897	0.1847	0.1999	0.2301	1.0000
Euro	Call	0.0514	0.0030	0.0346	0.0232	0.0425	0.1548
	Put	0.1601	0.1687	0.1732	0.1651	0.1782	0.8452
		0.2114	0.1717	0.2078	0.1883	0.2207	1.0000
Japanese yen	Call	0.1216	0.1024	0.0947	0.1033	0.0882	0.5102
	Put	0.1000	0.0873	0.0848	0.0783	0.1393	0.4898
		0.2216	0.1897	0.1795	0.1816	0.2275	1.0000
Swiss franc	Call	0.1353	0.1100	0.1113	0.1159	0.1420	0.6144
	Put	0.0738	0.0789	0.0697	0.0771	0.0862	0.3856
		0.2091	0.1888	0.1809	0.1930	0.2281	1.0000

Notes: $w_{i,t-1}$, $w_{i,t-2}$, $w_{i,t-3}$, $w_{i,t-4}$ and $w_{i,t-5}$ represent weights corresponding to the previous day 1, day 2, day 3, day 4 and day 5, respectively.

Table 5: Weights Vector for Full Sample: MAE Objective Function

Currency	Options (i)	Weights corresponding to previous day 1 to day 5					Total Weight
		$w_{i,t-1}$	$w_{i,t-2}$	$w_{i,t-3}$	$w_{i,t-4}$	$w_{i,t-5}$	
British pound	Call	0.1235	0.1229	0.1288	0.1282	0.1489	0.6522
	Put	0.0586	0.0849	0.0647	0.0682	0.0714	0.3478
		0.1820	0.2078	0.1935	0.1963	0.2204	1.0000
Euro	Call	0.0885	0.0799	0.1377	0.1113	0.1134	0.5308
	Put	0.1059	0.0906	0.0622	0.1005	0.1100	0.4692
		0.1944	0.1705	0.1999	0.2118	0.2234	1.0000
Japanese yen	Call	0.1003	0.0741	0.0760	0.0728	0.0647	0.3878
	Put	0.1117	0.1115	0.1204	0.1313	0.1372	0.6122
		0.2120	0.1856	0.1964	0.2041	0.2019	1.0000
Swiss franc	Call	0.1500	0.1056	0.1052	0.1206	0.1369	0.6183
	Put	0.0634	0.0693	0.0844	0.0858	0.0787	0.3817
		0.2134	0.1749	0.1896	0.2065	0.2157	1.0000

Notes: $w_{i,t-1}$, $w_{i,t-2}$, $w_{i,t-3}$, $w_{i,t-4}$ and $w_{i,t-5}$ represent weights corresponding to the previous day 1, day 2, day 3, day 4 and day 5, respectively.

Table 6: Weights Vector for Full Sample:
MAPE Objective Function

Currency	Options (i)	Weights corresponding to previous day 1 to day 5					Total Weight
		$w_{i,t-1}$	$w_{i,t-2}$	$w_{i,t-3}$	$w_{i,t-4}$	$w_{i,t-5}$	
British pound	Call	0.0872	0.1018	0.1028	0.0730	0.1178	0.4827
	Put	0.1067	0.0987	0.0929	0.1138	0.1052	0.5173
		0.1939	0.2006	0.1956	0.1868	0.2231	1.0000
Euro	Call	0.0679	0.0989	0.1470	0.1287	0.1288	0.5714
	Put	0.1178	0.0710	0.0487	0.0952	0.0959	0.4286
		0.1858	0.1699	0.1956	0.2240	0.2247	1.0000
Japanese yen	Call	0.0987	0.0777	0.0760	0.0773	0.0756	0.4053
	Put	0.1109	0.1051	0.1202	0.1295	0.1291	0.5947
		0.2096	0.1828	0.1962	0.2068	0.2047	1.0000
Swiss franc	Call	0.1404	0.0820	0.0791	0.1217	0.1511	0.5743
	Put	0.0715	0.0871	0.1065	0.0941	0.0664	0.4257
		0.2119	0.1691	0.1856	0.2159	0.2175	1.0000

Notes: $w_{i,t-1}$, $w_{i,t-2}$, $w_{i,t-3}$, $w_{i,t-4}$ and $w_{i,t-5}$ represent weights corresponding to the previous day 1, day 2, day 3, day 4 and day 5, respectively.

To test the out-of-sample fit of the OV model, the weights (reported in Tables 4, 5 and 6) need to be recalculated by using equations (14), (15) and (16), respectively. Using the first 1000 observations, the estimated weights under MSE as objective function are presented in Table 7. As can be seen, the weighing patterns are qualitatively similar to those in the Table 4 for the full sample. The weights, under MAE and MAPE as objective functions, are also estimated and results presented in Tables 8 and 9, respectively. The weights in Table 8 are consistent with those reported in Table 5 for the full sample. Similar results can be seen in Table 9, which compares reasonably with those in Table 6 (full sample) for all currencies. Overall, the volatility weights obtained from the first 1000 observations to forecast options prices for out-of-sample test is fairly consistent with the volatility weights estimated from the full sample.

Next, using these new weights, the OV model price volatilities are recalculated for the first 1000 observations, which are then used to generate the forecast values for the remainder of the sample under this model. Similarly, for IV model (under MEM) and GV model, volatility points are recalculated for the first 1000 observations,

Table 7: Weights Vector for First 1000 Observations: MSE Objective Function

Currency	Options (i)	Weights corresponding to previous day 1 to day 5					Total Weight
		$w_{i,t-1}$	$w_{i,t-2}$	$w_{i,t-3}$	$w_{i,t-4}$	$w_{i,t-5}$	
British pound	Call	0.1312	0.1394	0.1262	0.1388	0.1592	0.6949
	Put	0.0748	0.0501	0.0523	0.0566	0.0714	0.3051
		0.2060	0.1895	0.1785	0.1954	0.2306	1.0000
Euro	Call	0.0378	0.0000	0.0213	0.0069	0.0420	0.1081
	Put	0.1795	0.1673	0.1903	0.1714	0.1833	0.8919
		0.2173	0.1673	0.2116	0.1783	0.2254	1.0000
Japanese yen	Call	0.1072	0.0796	0.0714	0.0743	0.0648	0.3972
	Put	0.1087	0.1020	0.1015	0.1140	0.1765	0.6028
		0.2159	0.1815	0.1729	0.1883	0.2413	1.0000
Swiss franc	Call	0.1419	0.1139	0.1094	0.1148	0.1470	0.6269
	Put	0.0707	0.0765	0.0616	0.0768	0.0874	0.3731
		0.2126	0.1904	0.1710	0.1916	0.2344	1.0000

Notes: $w_{i,t-1}$, $w_{i,t-2}$, $w_{i,t-3}$, $w_{i,t-4}$ and $w_{i,t-5}$ represent weights corresponding to the previous day 1, day 2, day 3, day 4 and day 5, respectively.

Table 8: Weights Vector for First 1000 Observations: MAE Objective Function

Currency	Options (i)	Weights corresponding to previous day 1 to day 5					Total Weight
		$w_{i,t-1}$	$w_{i,t-2}$	$w_{i,t-3}$	$w_{i,t-4}$	$w_{i,t-5}$	
British pound	Call	0.1443	0.1396	0.1566	0.1552	0.1758	0.7715
	Put	0.0558	0.0682	0.0247	0.0382	0.0416	0.2285
		0.2001	0.2078	0.1813	0.1934	0.2175	1.0000
Euro	Call	0.0804	0.0549	0.1175	0.0694	0.1139	0.4362
	Put	0.1279	0.0978	0.0858	0.1237	0.1286	0.5638
		0.2083	0.1527	0.2034	0.1932	0.2424	1.0000
Japanese yen	Call	0.0863	0.0314	0.0516	0.0520	0.0475	0.2687
	Put	0.1366	0.1508	0.1340	0.1511	0.1588	0.7313
		0.2228	0.1821	0.1856	0.2031	0.2063	1.0000
Swiss franc	Call	0.1725	0.1054	0.1083	0.1190	0.1390	0.6442
	Put	0.0634	0.0671	0.0549	0.0887	0.0817	0.3558
		0.2359	0.1724	0.1632	0.2077	0.2207	1.0000

Notes: $w_{i,t-1}$, $w_{i,t-2}$, $w_{i,t-3}$, $w_{i,t-4}$ and $w_{i,t-5}$ represent weight of previous day 1, day 2, day 3, day 4 and day 5, respectively.

Table 9: Weights Vector for First 1000 Observations: MAPE Objective Function

Currency	Options (i)	Weights corresponding to previous day 1 to day 5					Total Weight
		$w_{i,t-1}$	$w_{i,t-2}$	$w_{i,t-3}$	$w_{i,t-4}$	$w_{i,t-5}$	
British pound	Call	0.0929	0.1239	0.1160	0.0776	0.1245	0.5349
	Put	0.1019	0.0863	0.0767	0.1008	0.0994	0.4651
		0.1949	0.2101	0.1927	0.1784	0.2239	1.0000
Euro	Call	0.0549	0.0801	0.1195	0.0928	0.1297	0.4770
	Put	0.1420	0.0849	0.0821	0.1048	0.1092	0.5230
		0.1969	0.1650	0.2016	0.1976	0.2390	1.0000
Japanese yen	Call	0.0769	0.0365	0.0459	0.0554	0.0559	0.2706
	Put	0.1532	0.1442	0.1300	0.1470	0.1550	0.7294
		0.2301	0.1807	0.1759	0.2023	0.2109	1.0000
Swiss franc	Call	0.1410	0.0827	0.0858	0.1204	0.1528	0.5826
	Put	0.0797	0.0815	0.0920	0.0982	0.0660	0.4174
		0.2207	0.1642	0.1777	0.2185	0.2188	1.0000

Notes: $w_{i,t-1}$, $w_{i,t-2}$, $w_{i,t-3}$, $w_{i,t-4}$ and $w_{i,t-5}$ represent weights corresponding to the previous day 1, day 2, day 3, day 4 and day 5, respectively.

which are then used to generate the forecast values for the remainder of the sample under these models.

The estimated values of OVPE, under MSE as the objective function are first compared with GVPE, and the results are given in Table 10. As can be seen in the last column, the values of OVPE are systematically and considerably smaller than those of GVPE by all measures (MSE, MAE and MAPE). It indicates that the OV model performs better than the GV model in forecasting option prices volatility for all four currencies. Table 11 and Table 12 in which MAE and MAPE were used as objective functions, respectively, provide similar results as reported in Table 10.

The out-of-sample performance of OV model is then compared with that of IV model under MEM. Table 13 gives the results with MSE as the objective function. As can be seen, OVPE does extremely well compared to IVPE with MSE as the test criterion. The MAE and MAPE results in Table 13 are mixed. However, the forecast errors are now more in favor of OVPE compared to those observed in Tables 1 to 3 for the in-sample fit. Table 14 and Table 15 use MAE and MAPE, respectively, as objective functions and provide very similar results as reported in Table 13.

Table 10: Comparison of OVPE and GVPE (Out-of-Sample): MSE Objective Function

Measures	Currency	Options	OVPE	GVPE	$\frac{\text{OVPE} - \text{GVPE}}{\text{GVPE}}\%$	Average difference %
MSE	British pound	Call	0.0631	0.1371	-53.98	-54.62
		Put	0.0711	0.1589	-55.25	
	Euro	Call	0.0520	0.2239	-76.78	-72.30
		Put	0.0425	0.1321	-67.83	
	Japanese yen	Call	0.0252	0.0398	-36.68	-26.31
		Put	0.0190	0.0226	-15.93	
	Swiss franc	Call	0.0213	0.0240	-11.25	-16.76
		Put	0.0192	0.0247	-22.27	
MAE	British pound	Call	0.6407	3.4086	-81.20	-59.24
		Put	0.6889	1.0983	-37.28	
	Euro	Call	0.5833	1.3180	-55.74	-50.48
		Put	0.5323	0.9717	-45.22	
	Japanese yen	Call	0.3998	0.5084	-21.36	-4.43
		Put	0.3215	0.2858	12.49	
	Swiss franc	Call	0.3713	0.3957	-6.17	-9.06
		Put	0.3773	0.4285	-11.95	
MAPE	British pound	Call	0.4388	0.7263	-39.58	-40.05
		Put	0.4040	0.6791	-40.51	
	Euro	Call	0.4779	1.1239	-57.48	-52.59
		Put	0.4741	0.9064	-47.69	
	Japanese yen	Call	0.4137	0.5434	-23.87	-8.52
		Put	0.4477	0.4191	6.82	
	Swiss franc	Call	0.3824	0.4265	-10.34	-13.75
		Put	0.4783	0.5774	-17.16	

Notes: OVPE and IVPE represent optimal weighted volatility model pricing error and implied volatility model pricing error, respectively. MSE and MAE measures are to be divided by 1000 and 100, respectively. In the last column, the average negative and positive differences indicate that OVPE is less than GVPE and OVPE is more than GVPE, respectively, by reported percent.

Table 11: Comparison of OVPE and GVPE (Out-of-Sample): MAE Objective Function

Measures	Currency	Options	Model pricing errors and their difference in percentage			
			OVPE	GVPE	$\frac{OVPE - GVPE}{GVPE}\%$	Average difference %
MSE	British pound	Call	0.0650	0.1371	-52.59	-53.29
		Put	0.0731	0.1589	-54.00	
	Euro	Call	0.0462	0.2239	-79.37	-75.53
		Put	0.0374	0.1321	-71.69	
	Japanese yen	Call	0.0271	0.0398	-31.91	-19.49
		Put	0.0210	0.0226	-7.08	
	Swiss franc	Call	0.0213	0.0240	-11.25	-16.56
		Put	0.0193	0.0247	-21.86	
MAE	British pound	Call	0.6500	3.4086	-80.93	-58.69
		Put	0.6980	1.0983	-36.45	
	Euro	Call	0.5491	1.3180	-58.34	-53.45
		Put	0.4998	0.9717	-48.56	
	Japanese yen	Call	0.4091	0.5084	-19.53	-2.14
		Put	0.3294	0.2858	15.26	
	Swiss franc	Call	0.3714	0.3957	-6.14	-8.86
		Put	0.3789	0.4285	-11.58	
MAPE	British pound	Call	0.4447	0.7263	-38.77	-39.26
		Put	0.4092	0.6791	-39.74	
	Euro	Call	0.4545	1.1239	-59.56	-55.07
		Put	0.4479	0.9064	-50.58	
	Japanese yen	Call	0.4236	0.5434	-22.05	-6.19
		Put	0.4596	0.4191	9.66	
	Swiss franc	Call	0.3824	0.4265	-10.34	-13.62
		Put	0.4798	0.5774	-16.90	

Notes: OVPE and IVPE represent optimal weighted volatility model pricing error and implied volatility model pricing error, respectively. MSE and MAE measures are to be divided by 1000 and 100, respectively. In the last column, the average negative and positive differences indicate that OVPE is less than GVPE and OVPE is more than GVPE, respectively, by reported percent.

5. Conclusion

This paper provides a new approach for computing the volatility explicitly from the currency options market prices. The objective is to assess how well the past options market price would forecast the future one. The emphasis is on assessing the accuracy of the forecasts, rather than on how forecasts are formed. The paper introduces a novel framework (OV model) to forecast future prices by finding an optimal linear combination of past absolute returns by minimizing different objectives functions (MSE, MAE, MAPE). The forecast performance of OV model is

Table 12: Comparison of OVPE and GVPE (Out-of-Sample): MAPE Objective Function

Measures	Currency	Options	OVPE	GVPE	$\frac{\text{OVPE} - \text{GVPE}}{\text{GVPE}}\%$	Average difference %
MSE	British pound	Call	0.0603	0.1371	-56.02	-56.64
		Put	0.0679	0.1589	-57.27	
	Euro	Call	0.0459	0.2239	-79.50	-75.59
		Put	0.0374	0.1321	-71.69	
	Japanese yen	Call	0.0270	0.0398	-32.16	-19.84
		Put	0.0209	0.0226	-7.52	
	Swiss franc	Call	0.0211	0.0240	-12.08	-17.38
		Put	0.0191	0.0247	-22.67	
MAE	British pound	Call	0.6244	3.4086	-81.68	-60.21
		Put	0.6728	1.0983	-38.74	
	Euro	Call	0.5451	1.3180	-58.64	-53.79
		Put	0.4961	0.9717	-48.95	
	Japanese yen	Call	0.4087	0.5084	-19.61	-2.13
		Put	0.3297	0.2858	15.36	
	Swiss franc	Call	0.3698	0.3957	-6.55	-9.20
		Put	0.3777	0.4285	-11.86	
MAPE	British pound	Call	0.4283	0.7263	-41.03	-41.37
		Put	0.3959	0.6791	-41.70	
	Euro	Call	0.4509	1.1239	-59.88	-55.38
		Put	0.4452	0.9064	-50.88	
	Japanese yen	Call	0.4233	0.5434	-22.10	-6.16
		Put	0.4601	0.4191	9.78	
	Swiss franc	Call	0.3815	0.4265	-10.55	-13.76
		Put	0.4794	0.5774	-16.97	

Notes: OVPE and IVPE represent optimal weighted volatility model pricing error and implied volatility model pricing error, respectively. MSE and MAE measures are to be divided by 1000 and 100, respectively. In the last column, the average negative and positive differences indicate that OVPE is less than GVPE and OVPE is more than GVPE, respectively, by reported percent.

Table 13: Comparison of OVPE and IVPE (Out-of-Sample): MSE Objective Function

Measures	Currency	Options	Model pricing errors and their difference in percentage			
			OVPE	IVPE	$\frac{\text{OVPE} - \text{IVPE}}{\text{IVPE}}\%$	Average difference %
MSE	British pound	Call	0.0631	0.4303	-85.34	-85.25
		Put	0.0711	0.4793	-85.17	
	Euro	Call	0.0520	0.2689	-80.66	-80.49
		Put	0.0425	0.2159	-80.31	
	Japanese yen	Call	0.0252	0.2238	-88.74	-83.11
		Put	0.0190	0.0844	-77.49	
	Swiss franc	Call	0.0213	0.1598	-86.67	-84.80
		Put	0.0192	0.1125	-82.93	
MAE	British pound	Call	0.6407	0.6108	4.90	2.07
		Put	0.6889	0.6941	-0.75	
	Euro	Call	0.5833	0.5569	4.74	6.84
		Put	0.5323	0.4886	8.94	
	Japanese yen	Call	0.3998	0.5970	-33.03	-9.91
		Put	0.3215	0.2840	13.20	
	Swiss franc	Call	0.3713	0.3779	-1.75	9.00
		Put	0.3773	0.3151	19.74	
MAPE	British pound	Call	0.4388	0.3959	10.84	6.08
		Put	0.4040	0.3987	1.33	
	Euro	Call	0.4779	0.4445	7.51	7.98
		Put	0.4741	0.4372	8.44	
	Japanese yen	Call	0.4137	0.5982	-30.84	-9.14
		Put	0.4477	0.3977	12.57	
	Swiss franc	Call	0.3824	0.3865	-1.06	11.15
		Put	0.4783	0.3877	23.37	

Notes: OVPE and IVPE represent optimal weighted volatility model pricing error and implied volatility model pricing error, respectively. MSE and MAE measures are to be divided by 1000 and 100, respectively. In the last column, the average negative and positive differences indicate that OVPE is less than IVPE and OVPE is more than IVPE, respectively, by reported percent.

then compared to the forecast performance of Engle's (2002) multiplicative error model for IV and a GARCH (1,1) model. Overall, the results indicate that the proposed OV model in this paper is capable of producing reasonably accurate forecasts for the put and call prices.

The empirical results of this paper have important implications for option traders who need to use forecasting model for options valuation purposes. The main contribution of this paper is to provide a general framework that can be easily implemented in spreadsheet applications. More accurate formulae would require

Table 14: Comparison of OVPE and IVPE (Out-of-Sample): MAE Objective Function

Measures	Currency	Options	Model pricing errors and their difference in percentage			
			OVPE	IVPE	$\frac{\text{OVPE} - \text{IVPE}}{\text{IVPE}}\%$	Average difference %
MSE	British pound	Call	0.0650	0.4303	-84.89	-84.82
		Put	0.0731	0.4793	-84.75	
	Euro	Call	0.0462	0.2689	-82.82	-82.75
		Put	0.0374	0.2159	-82.68	
	Japanese yen	Call	0.0271	0.2238	-87.89	-81.50
		Put	0.0210	0.0844	-75.12	
	Swiss franc	Call	0.0213	0.1598	-86.67	-84.76
		Put	0.0193	0.1125	-82.84	
MAE	British pound	Call	0.6500	0.6108	6.42	3.49
		Put	0.6980	0.6941	0.56	
	Euro	Call	0.5491	0.5569	-1.40	0.45
		Put	0.4998	0.4886	2.29	
	Japanese yen	Call	0.4091	0.5970	-31.47	-7.74
		Put	0.3294	0.2840	15.99	
	Swiss franc	Call	0.3714	0.3779	-1.72	9.26
		Put	0.3789	0.3151	20.25	
MAPE	British pound	Call	0.4447	0.3959	12.33	7.48
		Put	0.4092	0.3987	2.63	
	Euro	Call	0.4545	0.4445	2.25	2.35
		Put	0.4479	0.4372	2.45	
	Japanese yen	Call	0.4236	0.5982	-29.19	-6.81
		Put	0.4596	0.3977	15.56	
	Swiss franc	Call	0.3824	0.3865	-1.06	11.35
		Put	0.4798	0.3877	23.76	

Notes: OVPE and IVPE represent optimal weighted volatility model pricing error and implied volatility model pricing error, respectively. MSE and MAE measures are to be divided by 1000 and 100, respectively. In the last column, the average negative and positive differences indicate that OVPE is less than IVPE and OVPE is more than IVPE, respectively, by reported percent.

Table 15: Comparison of OVPE and IVPE (Out-of-Sample): MAPE Objective Function

Measures	Currency	Options	OVPE	IVPE	$\frac{\text{OVPE} - \text{IVPE}}{\text{IVPE}}$ %	Average difference %
			Model pricing errors and their difference in percentage			
MSE	British pound	Call	0.0603	0.4303	-85.99	-85.91
		Put	0.0679	0.4793	-85.83	
	Euro	Call	0.0459	0.2689	-82.93	-82.80
		Put	0.0374	0.2159	-82.68	
	Japanese yen	Call	0.0270	0.2238	-87.94	-81.59
		Put	0.0209	0.0844	-75.24	
	Swiss franc	Call	0.0211	0.1598	-86.80	-84.91
		Put	0.0191	0.1125	-83.02	
MAE	British pound	Call	0.6244	0.6108	2.23	-0.42
		Put	0.6728	0.6941	-3.07	
	Euro	Call	0.5451	0.5569	-2.12	-0.29
		Put	0.4961	0.4886	1.53	
	Japanese yen	Call	0.4087	0.5970	-31.54	-7.72
		Put	0.3297	0.2840	16.09	
	Swiss franc	Call	0.3698	0.3779	-2.14	8.86
		Put	0.3777	0.3151	19.87	
MAPE	British pound	Call	0.4283	0.3959	8.18	3.74
		Put	0.3959	0.3987	-0.70	
	Euro	Call	0.4509	0.4445	1.44	1.63
		Put	0.4452	0.4372	1.83	
	Japanese yen	Call	0.4233	0.5982	-29.24	-6.77
		Put	0.4601	0.3977	15.69	
	Swiss franc	Call	0.3815	0.3865	-1.29	11.18
		Put	0.4794	0.3877	23.65	

Notes: OVPE and IVPE represent optimal weighted volatility model pricing error and implied volatility model pricing error, respectively. MSE and MAE measures are to be divided by 1000 and 100, respectively. In the last column, the average negative and positive differences indicate that OVPE is less than IVPE and OVPE is more than IVPE, respectively, by reported percent.

solving quadratic or higher order algebra equations, for which no simple closed-form solutions can be obtained. The model proposed in this paper is simple and robust relative to MEM and GARCH(1,1) for forecasting option price. This model is also flexible as it can be modified to accommodate different objective functions to forecast future volatility with different option pricing models. In future research, it will be interesting to compare this approach with other stochastic models where the implied volatility is updated daily. A good estimation of future volatility surface across strike prices is also another possible area of future research (see, for example, Klebaner, Le and Lipster, 2006).

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